

# Magnetic Charge As a "Hidden" Gauge Symmetry

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*Received February 22, 1994*

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A theory containing both electric and magnetic charges is formulated using two vectors potentials,  $A^\mu$  and  $C^\mu$ . This has the aesthetic advantage of treating electric and magnetic charges both as gauge symmetries, but it has the experimental disadvantage of introducing a second massless gauge boson (the "magnetic" photon) which is not observed. This problem is dealt with by using the Higgs mechanism to give a mass to one of the gauge bosons while the other remains massless. This effectively "hides" the magnetic charge, and the symmetry associated with it, when one is at an energy scale far enough removed from the scale of the symmetry breaking.

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## 1. INTRODUCTION

Since the seminal work of Dirac (1931, 1948) magnetic monopoles have excited much theoretical interest, but there has been no confirmed experimental evidence of their existence up to the present. Dirac's formulation requires the introduction of a singular vector potential so that the definition  $\mathbf{B} = \nabla \times \mathbf{A}$ , may be used, while still having  $\nabla \cdot \mathbf{B} = \rho_m$ , where  $\rho_m$  is the magnetic charge density. The vector potential  $\mathbf{A}$  is singular along a line which runs from the magnetic charge off to spatial infinity. By requiring that the string singularity have no physical effect (i.e., the wavefunction of a charged particle must vanish along it), one arrives at Dirac's condition for the quantization of electric charge. A fiber-bundle formulation of magnetic monopoles has been given by Wu and Yang (1975) which avoids the need for a singular vector potential, but defines the vector potential differently in two different regions surrounding the magnetic charge. The two vector potentials are related by a gauge transformation, and requiring that the gauge transformation function be single-valued yields the Dirac quantization condition again.

This paper is dedicated to my grandparents Herbert and Anneliese Schmidt.

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In this article we present a different formulation of a magnetic charge based upon the gauge principle. Electric charge is a gauge charge which is coupled to a gauge field, the photon, by replacing the ordinary derivative with the gauge-covariant derivative in the Lagrangian. In electrodynamics the gauge field corresponds to the four-vector potential  $A_\mu$ . Since the generalized Maxwell equations with electric and magnetic charges appear symmetric between the two types of charge and currents, one might ask if it is possible to treat magnetic charge, like electric charge, as a gauge symmetry. The gauge principle then implies that there must be a second massless gauge boson corresponding to magnetic charge. In the next section we will show that Maxwell's equations with electric and magnetic charges naturally have room for two four-vector potentials  $A_\mu$  and  $C_\mu$ . One is then left with two massless photons, while only one massless photon is known to exist experimentally. This difficulty can be overcome through the use of the Higgs mechanism (Higgs, 1964*a,b*, 1966), which allows one to make gauge bosons massive while not violating the gauge invariance of the theory. This allows the charge which is coupled to the massive gauge boson to remain "hidden" as long as one is not too near the energy scale of the symmetry breaking.

There have been previous attempts to construct a field theory of magnetic charges in terms of a pair of four-potentials by Cabibbo and Ferrari (1962) and Zwanziger (1971), but the second vector potential has always been somewhat problematic since then there are apparently too many degrees of freedom. Spontaneous symmetry breaking allows one to deal with these extra degrees of freedom in a natural way.

## 2. GENERALIZED MAXWELL EQUATIONS AND DUALITY

The generalized Maxwell equations in the presence of electric and magnetic charges and currents are (Jackson, 1975)

$$\begin{aligned} \nabla \cdot \mathbf{E} = \rho_e & & \nabla \times \mathbf{B} = \frac{1}{c} \left( \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_e \right) \\ \nabla \cdot \mathbf{B} = \rho_m & & -\nabla \times \mathbf{E} = \frac{1}{c} \left( \frac{\partial \mathbf{B}}{\partial t} + \mathbf{J}_m \right) \end{aligned} \quad (1)$$

These equations are invariant under the following duality transformation:

$$\begin{aligned} \mathbf{E} &\rightarrow \mathbf{E} \cos \theta + \mathbf{B} \sin \theta \\ \mathbf{B} &\rightarrow -\mathbf{E} \sin \theta + \mathbf{B} \cos \theta \end{aligned} \quad (2)$$

$$\begin{aligned} \rho_e &\rightarrow \rho_e \cos \theta + \rho_m \sin \theta, & \rho_m &\rightarrow -\rho_e \sin \theta + \rho_m \cos \theta \\ \mathbf{J}_e &\rightarrow \mathbf{J}_e \cos \theta + \mathbf{J}_m \sin \theta & \mathbf{J}_m &\rightarrow -\mathbf{J}_e \sin \theta + \mathbf{J}_m \cos \theta \end{aligned} \quad (3)$$

Now one can introduce two four-vector potentials  $A^\mu = (\phi_e, \mathbf{A})$  and  $C^\mu = (\phi_m, \mathbf{C})$  and write the  $\mathbf{E}$  and  $\mathbf{B}$  fields as

$$\begin{aligned}\mathbf{E} &= -\nabla\phi_e - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \nabla\times\mathbf{C} \\ \mathbf{B} &= -\nabla\phi_m - \frac{1}{c}\frac{\partial\mathbf{C}}{\partial t} + \nabla\times\mathbf{A}\end{aligned}\quad (4)$$

In electrodynamics with only electric charge  $\mathbf{E}$  consists of only the first two terms, while  $\mathbf{B}$  consists of only the last term. Taking  $\phi_e$  as a scalar and  $\mathbf{A}$  as a vector under spatial inversion leads to  $\mathbf{E}$  being a vector and  $\mathbf{B}$  being a pseudovector under spatial inversion. In order for  $\mathbf{E}$  to remain a vector and  $\mathbf{B}$  to remain a pseudovector in equation (4),  $\phi_m$  must be a pseudoscalar and  $\mathbf{C}$  a pseudovector. Using the gauge freedom which is possessed by the potentials, one can choose  $A^\mu$  and  $C^\mu$  such that they satisfy the Lorentz gauge condition

$$\begin{aligned}\partial_\mu A^\mu &= \frac{1}{c}\frac{\partial\phi_e}{\partial t} + \nabla\cdot\mathbf{A} = 0 \\ \partial_\mu C^\mu &= \frac{1}{c}\frac{\partial\phi_m}{\partial t} + \nabla\cdot\mathbf{C} = 0\end{aligned}\quad (5)$$

On substituting the expressions for  $\mathbf{E}$  and  $\mathbf{B}$  in terms of the potentials (4) into the generalized Maxwell equations (1), using the two Lorentz gauge conditions (5), and applying some standard vector calculus identities ( $\nabla\cdot[\nabla\times\mathbf{a}] = 0$ ,  $\nabla\times[\nabla\phi] = 0$ , and  $\nabla\times[\nabla\times\mathbf{a}] = \nabla[\nabla\cdot\mathbf{a}] - \nabla^2\mathbf{a}$ ), one arrives at the following alternative form for the equations:

$$\begin{aligned}\nabla^2\phi_e - \frac{1}{c^2}\frac{\partial^2\phi_e}{\partial t^2} &= -\rho_e & \nabla^2\phi_m - \frac{1}{c^2}\frac{\partial^2\phi_m}{\partial t^2} &= -\rho_m \\ \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} &= -\frac{1}{c}\mathbf{J}_e & \nabla^2\mathbf{C} - \frac{1}{c^2}\frac{\partial^2\mathbf{C}}{\partial t^2} &= -\frac{1}{c}\mathbf{J}_m\end{aligned}\quad (6)$$

From here on we will set  $c = 1$ . Since  $\phi_m$  is a pseudoscalar and  $\mathbf{C}$  a pseudovector, equations (6) imply that the magnetic charge density  $\rho_m$  and magnetic current density  $\mathbf{J}_m$  are pseudoscalars and pseudovectors, respectively. The significance of (6) is that it demonstrates that Maxwell's equations with both electric and magnetic charges naturally allow for two four-vector potentials (i.e., two "photons"). It is desirable to cast our results to this point in covariant notation in terms of the two four-potentials  $A^\mu$  and  $C^\mu$  in order to be able to obtain a Lagrangian density for the generalized Maxwell equations. Defining two field strength tensors in terms of the two four-vector potentials

$$\begin{aligned} F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu \\ G^{\mu\nu} &= \partial^\mu C^\nu - \partial^\nu C^\mu \end{aligned} \quad (7)$$

one can write the Maxwell equations with magnetic charge, equations (6), in the following covariant form:

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= \partial_\mu \partial^\mu A^\nu = J_e^\nu \\ \partial_\mu G^{\mu\nu} &= \partial_\mu \partial^\mu C^\nu = J_m^\nu \end{aligned} \quad (8)$$

where the Lorentz gauge of equation (5) has been used.

The duality transformations (2) and (3) can now be written in terms of the four-potentials and four-currents

$$A^\mu \rightarrow A^\mu \cos \theta + C^\mu \sin \theta \quad (9)$$

$$C^\mu \rightarrow -A^\mu \sin \theta + C^\mu \cos \theta$$

$$J_e^\mu \rightarrow J_e^\mu \cos \theta + J_m^\mu \sin \theta \quad (10)$$

$$J_m^\mu \rightarrow -J_e^\mu \sin \theta + J_m^\mu \cos \theta$$

Finally the **E** and **B** fields of (4) can be written in terms of the field-strength tensors as

$$\begin{aligned} E_i &= F^{i0} + \frac{1}{2} \varepsilon^{ijk} G_{jk} = F^{i0} - \mathcal{G}^{i0} \\ B_i &= G^{i0} - \frac{1}{2} \varepsilon^{ijk} F_{jk} = G^{i0} + \mathcal{F}^{i0} \end{aligned} \quad (11)$$

where

$$\begin{aligned} \mathcal{F}^{\mu\nu} &= \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \\ \mathcal{G}^{\mu\nu} &= \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} G_{\alpha\beta} \end{aligned} \quad (12)$$

$\varepsilon^{\mu\nu\rho\sigma}$  is the totally antisymmetric fourth-rank Levi-Civita tensor, where  $\varepsilon^{0123} = +1$  with even permutations of the indices giving +1 and odd permutations giving -1. Here  $\mathcal{F}^{\mu\nu}$  and  $\mathcal{G}^{\mu\nu}$  are the duals of  $F^{\mu\nu}$  and  $G^{\mu\nu}$ . From (11) it looks as if two ‘‘photons’’ are contributing to the **E** and **B** fields. However, using spontaneous symmetry breaking via a scalar field, one of the ‘‘photons’’ is made massive, thus effectively reducing the above definitions to their usual form of  $E_i = F^{i0}$  and  $B_i = \frac{1}{2} \varepsilon^{ijk} F_{jk}$  as long as one is not too close to the energy scalar of the breaking.

It is now straightforward to write down a Lagrangian density which gives the Maxwell equations (8). These equations are simply two copies of

the same equation with the source of one being an electric four-current  $J_e^\mu$  and the source of the other being a magnetic four-current  $J_m^\mu$ . Thus the most obvious thing to do is to add a new kinetic term and a new source term for the four-potential  $C^\mu$ ,

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - J_e^\mu A_\mu - J_m^\mu G_\mu \quad (13)$$

Variation of this Lagrange density with respect to  $A^\mu$  and  $C^\mu$  yields the field equations (8) in the Lorentz gauge. This Lagrangian is invariant under the duality transformation of (9), (10). This property will be used in the next section. Also the Lagrangian is a scalar despite the fact that it contains pseudoquantities ( $J_m^\mu$  and  $C_\mu$ ), since these quantities only occur in combinations which result in real scalars under parity.

One drawback to this simple extension of the usual Maxwell Lagrangian is that it does not yield the expected energy-momentum tensor in terms of the  $\mathbf{E}$  and  $\mathbf{B}$  fields as defined in (11). In particular, one would expect there to be cross terms between the field-strength tensors  $F^{\mu\nu}$  and  $G^{\mu\nu}$  coming, for example, from  $T^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$  with the use of (11). In addition (13), while giving the correct Maxwell equations, makes it appear as if the electric and magnetic four-currents are completely decoupled from one another, contrary to physical intuition that an electric charge and magnetic charge would interact with one another. However, the Lagrange density and energy-momentum tensor are not unique. One is free to add any four-divergence to the Lagrangian without changing the Maxwell equations, and one is also free to add any four-divergence with the proper antisymmetry property (Goldstein, 1980) without changing the conservation laws or integral quantities associated with the energy-momentum tensor. By adding  $+\frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}G_{\rho\sigma}$  to the Lagrangian one mixes the two "photons" and obtains the usual energy-momentum tensor with all the cross terms that are implied by (11). Adding  $\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}G_{\rho\sigma}$  to the Lagrangian does not change the Maxwell equations (8) obtained from the Lagrangian, since it is a total four-divergence by the antisymmetry property of  $\varepsilon^{\mu\nu\rho\sigma}$ . In this paper we will not explicitly write this cross term in the Lagrangian, since all we require for the present development is the minimal Lagrangian that yields the Maxwell equations.

### 3. THE SCALAR FIELD

The Lagrangian formulation of Maxwell's equations as expressed in (13) possesses the theoretically pleasing feature of treating electric and magnetic charge symmetrically, but has the experimentally displeasing feature of introducing a second massless gauge boson. This second gauge

boson (and the symmetry associated with it) can be “hidden” using the Higgs mechanism. First one introduces a complex scalar field  $\Phi$  which carries a scalar electric charge  $q_e$  and a pseudoscalar magnetic charge  $q_m$  and must be gauged with respect to both the four-vector and pseudo four-vector potentials ( $A^\mu$  and  $C^\mu$ ). The Lagrangian is

$$\mathcal{L}_S = (\partial_\mu + iq_e A_\mu + iq_m C_\mu)\Phi^*(\partial^\mu - iq_e A^\mu - iq_m C^\mu)\Phi - V(\Phi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \quad (14)$$

where  $\Phi^*$  is the complex conjugate of  $\Phi$ . The form of the gauge-covariant derivative in equation (14) is that of Zwanziger (1971) here applied to a scalar field rather than a spinor field. The electric and magnetic four-currents  $J_e^\mu$  and  $J_m^\mu$  of equation (13) can be written in terms of the scalar fields as

$$\begin{aligned} J_e^\mu &= iq_e[\Phi^*(D^\mu\Phi) - \Phi(D^\mu\Phi)^*] \\ J_m^\mu &= iq_m[\Phi^*(D^\mu\Phi) - \Phi(D^\mu\Phi)^*] \end{aligned} \quad (15)$$

where  $D^\mu = (\partial^\mu - iq_e A^\mu - iq_m C^\mu)$  is the gauge-covariant derivative. Since  $q_e$  and  $q_m$  are scalar and pseudoscalar quantities, respectively, it follows from (15) that  $J_e^\mu$  and  $J_m^\mu$  are a real four-current and a pseudo-four-current, respectively. The gauge group of the above Lagrangian is  $U(1) \times U(1)$  (Carmeli, 1982), which is not a semisimple group. This leads one to suspect that there will be no quantization condition between electric and magnetic charges. The potential  $V(\Phi^2)$  contains the usual mass and quartic self-interaction terms in order to develop a VEV,

$$V(\Phi^2) = m^2(\Phi^*\Phi) + \lambda(\Phi^*\Phi)^2 \quad (16)$$

The self-interaction coupling constant  $\lambda$  is taken to be positive definite, and for  $m^2 < 0$  the potential acquires a vacuum expectation value of

$$\langle\Phi\rangle = \left(\frac{-m^2}{2\lambda}\right)^{1/2} \equiv \frac{v}{\sqrt{2}} \quad (17)$$

Parametrizing the complex field in terms of real fields with the VEV chosen to lie along the real component yields

$$\begin{aligned} \Phi(x) &= \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \\ &\approx \frac{1}{\sqrt{2}}(v + \eta(x))e^{i\xi(x)/v} \end{aligned} \quad (18)$$

Now using the gauge freedom of the Lagrangian in (14), one can transform the  $\zeta(x)$  field away by making a gauge transformation to the unitary gauge

$$\begin{aligned}\Phi'(x) &= e^{-i\zeta(x)/v}\Phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \\ A'_\mu(x) &= A_\mu(x) - \frac{1}{2q_e v} \partial_\mu \zeta(x) \\ C'_\mu(x) &= C_\mu(x) - \frac{1}{2q_m v} \partial_\mu \zeta(x)\end{aligned}\quad (19)$$

$\mathcal{L}_S$  is invariant under these transformations. Substituting these unitary gauge fields into (14) yields

$$\mathcal{L}_S = \mathcal{L}_0 + \mathcal{L}_I \quad (20)$$

where  $\mathcal{L}_0$  contains the kinetic energy and mass terms

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(2m^2)\eta^2 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{4}G'_{\mu\nu}G'^{\mu\nu} \\ &+ \frac{1}{2}v^2(q_e^2 A'_\mu A'^\mu + q_m^2 C'_\mu C'^\mu + 2q_e q_m A'_\mu C'^\mu)\end{aligned}\quad (21)$$

where  $F'_{\mu\nu}$  and  $G'_{\mu\nu}$  indicate that the field-strength tensors are in terms of the gauge-transformed fields and  $C'_\mu$ . Note that both the field-strength tensors and therefore the equations of motion derived from them are invariant under the gauge transformations (19). The interaction terms of the Lagrangian  $\mathcal{L}_I$  are

$$\begin{aligned}\mathcal{L}_I &= -\lambda v v^3 - \frac{\lambda}{4}\eta^4 + v\eta(q_e^2 A'_\mu A'^\mu + q_m^2 C'_\mu C'^\mu + 2q_e q_m A'_\mu C'^\mu) \\ &+ \frac{\eta^2}{2}(q_e^2 A'_\mu A'^\mu + q_m^2 C'_\mu C'^\mu + 2q_e q_m A'_\mu C'^\mu)\end{aligned}\quad (22)$$

A disturbing feature in both equations (21) and (22) is the presence of the cross terms between  $A'_\mu$  and  $C'_\mu$ , which makes finding the mass spectrum of the gauge bosons after symmetry breaking complicated. Also it appears that both gauge bosons have become massive even though only one scalar degree of freedom has been absorbed. However, one can now use the freedom of the duality transformation (9) to diagonalize away the cross terms  $A'_\mu C'^\mu$  in both  $\mathcal{L}_0$  and  $\mathcal{L}_I$ . One can write the gauge boson mass terms and gauge boson-scalar interaction terms in the form of  $2 \times 2$  matrices,

$$K(A'_\mu C'_\mu) \begin{pmatrix} q_e^2 & q_e q_m \\ q_e q_m & q_m^2 \end{pmatrix} \begin{pmatrix} A'^\mu \\ C'^\mu \end{pmatrix} \quad (23)$$

where  $K = v^2/2$ ,  $v$ ,  $1/2$  for the gauge boson mass, trilinear interaction, and quartic interaction terms, respectively. Now applying a duality transformation as in (9) with  $\cos \theta = q_m/(q_e^2 + q_m^2)^{1/2}$  and  $\sin \theta = q_e/(q_e^2 + q_m^2)^{1/2}$  the mass matrix and interaction matrices are diagonalized

$$K(A''_\mu C''_\mu) \begin{pmatrix} 0 & 0 \\ 0 & (q_e^2 + q_m^2) \end{pmatrix} \begin{pmatrix} A''^\mu \\ C''^\mu \end{pmatrix} \quad (24)$$

The double primes indicate that these are the duality-rotated fields which are obtained from  $A'_\mu$  and  $C'_\mu$ , which are themselves the gauge-transformed fields of the original  $A_\mu$  and  $C_\mu$ . However, as the Lagrangian is invariant under both the gauge and duality transformations, we will from here on drop the primes. This diagonalization of the mass matrix by a duality rotation is the same procedure that is used in the standard model (Glashow, 1961; Weinberg, 1967; Salam, 1968) to obtain the physical mass spectrum of the electroweak gauge bosons. One potential problem is the cross term  $\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}G_{\rho\sigma}$ , which is gauge invariant, but is not invariant under the duality rotation. Since it is a total divergence, with no effect on the field equations, it could be added to the Lagrangian at any point [i.e., it could be added to the final Lagrangian, equation (25) below]. Alternatively, one could add it to the original Lagrangian (14) with a coefficient in front chosen so that after the duality rotation it would become  $\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}G_{\rho\sigma}$ . The original particle spectrum of two massless gauge bosons and two scalar field has, through the Higgs mechanism, become one massive scalar field, one massless gauge field, and one massive gauge field. Writing out the final form of the Lagrangian gives

$$\begin{aligned} \mathcal{L}_S = & \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(2m^2)\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \\ & + \frac{1}{2}g^2v^2C_\mu C^\mu + g^2vC_\mu C^\mu\eta + \frac{1}{4}g^2C_\mu C^\mu\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 \end{aligned} \quad (25)$$

where  $g = (q_e^2 + q_m^2)^{1/2}$  is the coupling strength of the scalar field to the gauge boson  $C_\mu$ . The scalar field  $\eta(x)$  can be said to carry a magnetic charge of strength  $g$ , whose associated symmetry is broken leading to the gauge boson connected with it,  $C_\mu$ , having a mass,  $m_C = gv$ . Therefore, the scalar  $\eta(x)$  will have a Yukawa field rather than a Coulomb field surrounding it, and the magnetic charge will not be detectable unless one probes down to distances of the order of  $r = 1/m_C$ . Notice that  $A_\mu$  is now completely decoupled from the scalar field  $\eta(x)$  and that the Lagrangian (25) only has one charge,  $g$ . This corresponds to the well-known result that if all particles in a theory have the same ratio of electric to magnetic charge it is always possible to use the duality rotation (10) to rotate away one of



the charges. In order to make both electric and magnetic charges play a nontrivial role, one could introduce a second complex scalar field  $\Phi_2(x)$  with no self-interaction and whose original couplings  $q'_e$  and  $q'_m$  are different from  $q_e$  and  $q_m$ . The field  $\Phi_2(x)$  would have to undergo a phase rotation, in conjunction with (19), in order to allow its Lagrangian to remain invariant. [This phase transformation would depend on  $q_e$ ,  $q_m$ ,  $q'_e$ ,  $q'_m$ ,  $v$ , and  $\zeta(x)$ , but the explicit expression is not given, since we will not pursue this point in detail here.] On performing the duality rotation, the gauge boson- $\Phi_2(x)$  interaction matrices would in general include couplings between both components of  $\Phi_2$  and both gauge bosons. Thus  $\Phi_2(x)$  would represent two real scalar fields carrying both electric and broken magnetic charges, whose values would depend on the arbitrary values  $q'_e$  and  $q'_m$ .

#### 4. DISCUSSION AND CONCLUSIONS

In this paper we have attempted to give a formulation of magnetic charge as a gauge symmetry which is "hidden" through spontaneous symmetry breaking. This was done through the introduction of two gauge potentials,  $A_\mu$  and  $C_\mu$ . The presence of two massless gauge bosons in the Lagrangian was resolved by introducing a complex scalar field  $\Phi(x)$  and using the Higgs mechanism to give a mass to one of the gauge bosons while leaving the other one massless. Thus the existence of the massive gauge boson and the symmetry associated with it was hidden as long as one was at an energy scale less than the mass of the gauge boson,  $m_C$ .

Although this formulation of magnetic charge appears to be very different from Dirac's (1931, 1948) approach, there is a certain correspondence: In Dirac's formulation the string is an extra, nonlocal degree of freedom which is "hidden" through the Dirac quantization condition; in the present formulation the "magnetic" photon is an extra, local degree of freedom which is "hidden" through the Higgs mechanism. One difference between this formulation and previous formulations, which involved either singular potentials (Dirac, 1931, 1948) or gauge potentials which are defined differently over different domains (Wu and Yang, 1975), is that we do not obtain a charge quantization condition. However, both grand unified gauge theories and Kaluza-Klein theories give alternative explanations for the quantization of charge, so this loss of an explanation for charge quantization might not be so unpleasant. The broken charge  $g = (q_e^2 + q_m^2)^{1/2}$  that is carried by  $\eta$  depends only on the unconstrained (at the classical level) values of  $q_e$  and  $q_m$ .

There is much arbitrariness in this formulation of a magnetic charge: the charge  $g$ , the mass of  $\eta$ , the mass of the gauge boson, and the self-coupling of  $\eta$  are all unspecified. The aim of this paper was not to

construct a fully realistic model of a physical monopole, but to give a treatment where magnetic charge is treated, initially, exactly as electric charge, and then provide an explanation for the nonobservation of the symmetry associated with the magnetic charge. An interesting possibility of this formulation is to use it to construct fermions out of bound states of scalar particles carrying a broken magnetic charge and scalar particles carrying an unbroken electric charge. If the bound state were bound to a radius smaller than  $r = 1/m_C$ , then the electric charge would “see” most of the broken magnetic charge. There would be an angular momentum associated with the bound state (Saha, 1936, 1949; Wilson, 1949) which would depend on  $g$ ,  $m_C$ , the electric charge, and the size of the bound state. (For a system with an unbroken magnetic charge and electric charge the angular momentum is independent of the distance between the two charges, but with a broken magnetic charge; the closer the electric charge got, the more magnetic charge it would “see”). The gauge couplings and symmetry breaking could then be adjusted so as to let one interpret the angular momentum of the charge–monopole system as the spin of the bound state. Finally it has been shown that a charge–monopole system obeys Fermi–Dirac statistics (Goldhaber, 1976). It is plausible to postulate that this result would still hold if the magnetic charge of the monopole were “hidden” through spontaneous symmetry breaking.

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